MAS348 – Mock Exam Solutions

1(i)(a) Both Alice and Bob have set of actions $\{0, 1, 2, 3, 4\}$ and the payoffs are given in tabular for as follows:

	0	1	2	3	4
0	0,0	0,1	0,2	0,3	0,4
1	1,0	1,1	1,2	1,3	1,3
2	2,0	2,1	2,2	2,2	2,2
3	3,0	3,1	2,2	2,2	3,1
4	4,0	3,1	2,2	1,3	2,2

1(i)(b) Alice's strategies 0 and 1 are dominated (and hence also weakly dominated) by strategy 2. Alice's strategy 2 is weakly dominated by strategy 3.

Similarly, Bob's strategies 0 and 1 are dominated (and hence also weakly dominated) by strategy 2. Bob's strategy 2 is weakly dominated by strategy 3.

1(i)(c) If we eliminate dominated strategies we do not lose Nash equilibria, and after repeated elimination of dominated strategies we end up with the following game

	2	3	4
2	2,2	2,2	2,2
3	2,2	2,2	3,1
4	2,2	1,3	2,2

Alice's best response to Bob's 2 is any of 2, 3, 4; her best response to 3 is any of 2, 3; her best response to 4 is 3. Similarly, Bob's best response to Alice's 2 is any of 2, 3, 4; his best response to 3 is any of 2, 3; his best response to 4 is 3. We underline best responses and obtain

	2	3	4
2	$\underline{2},\underline{2}$	$\underline{2},\underline{2}$	$2,\underline{2}$
3	$\underline{2},\underline{2}$	$\underline{2},\underline{2}$	$\underline{3},1$
4	$\underline{2},2$	$1,\underline{3}$	$2,2$

and we can read-off four Nash equilibria: (2,2), (2,3), (3,2), (3,3). These are also Nash equilibria of the original game.

1(i)(d) For example, had we eliminated Alice's weakly dominated strategy 2 we would have lost the two Nash equilibria (2,2) and (2,3).

1(ii)(a) Company 1 profits are given by $f(q_1, q_2) = (100 - 6q_1 - 2q_2 - 10)q_1$ and company 2 profits by $g(q_1, q_2) = (200 - 5q_1 - 7q_2 - 20)q_2$.

To find company 1 best response to company 2 production of q_2 we compute

$$\frac{\partial f}{\partial q_1} = -12q_1 - 2q_2 + 90,$$

set it to zero and solve for q_1, giving $q_1 = (45 - q_2)/6$.

To find company 2 best response to company 1 production of q_1 we compute

$$\frac{\partial g}{\partial q_2} = 180 - 5q_1 - 14q_2$$

set it to zero and solve for q_2, giving $q_2 = (180 - 5q_1)/14$.

1(ii)(b) A Nash equilibrium will occur at (q_1^*, q_2^*) precisely when $q_1^* = (45 - q_2^*)/6$ and $q_2^* = (180 - 5q_1^*)/14$. We solve this system of equations and obtain $q_1^* = 450/79$, $q_2^* = 855/79$.

2(i)(a) Since $\{\widehat{t} \mid t \in T\} \subseteq \Delta^C$, $\max_{p\in\Delta^R}\min_{q\in\Delta^C} u(p,q) \le \max_{p\in\Delta^R}\min_{t\in T} u(p,\widehat{t})$. To show the opposite inequality, fix a $p \in \Delta^R$ and pick $\tau \in T$ such that $\min_{t\in T} u(p,\widehat{t}) = u(p,\widehat{\tau})$. For any $q \in \Delta^C$, $u(p,q) = \sum_{t\in T} q(t)u(p,\widehat{t}) \ge \sum_{t\in T} q(t)u(p,\widehat{\tau}) = u(p,\widehat{\tau})$ and $\min_{q\in\Delta^C} u(p,q) \ge u(p,\widehat{\tau}) = \min_{t\in T} u(p,\widehat{t})$, and so $\max_{p\in\Delta^R}\min_{q\in\Delta^C} u(p,q) \ge \max_{p\in\Delta^R}\min_{t\in T} u(p,\widehat{t})$.

2(i)(b) p^* is the value of $p \in \Delta^R$ which maximizes $\min_{q\in\Delta^C} u(p,q)$ and by the previous lemma, it is the value of $p \in \Delta^R$ which maximizes $\min_{t\in T} u(p,\widehat{t})$ and that maximal value is V, hence $\min_{t\in T} u(p^*,\widehat{t}) = V$.

2(ii)(a) (Notice the correction in red in the mock exam.)

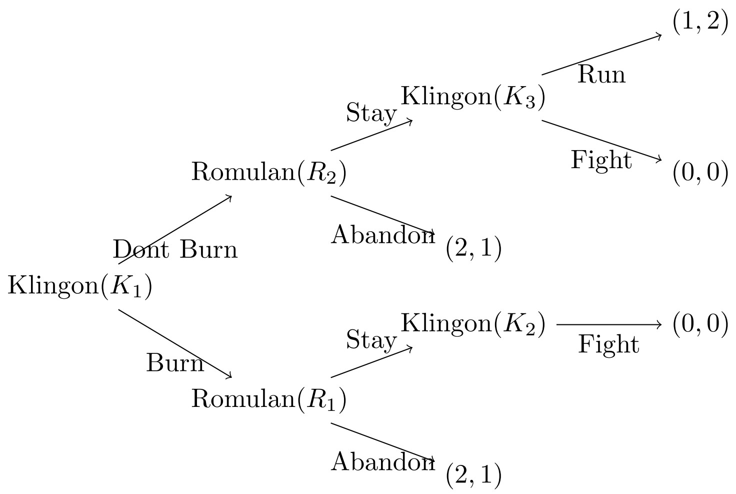
Assume Alice mixes I, II, II with probabilities p_1, p_2, p_3 and Bob mixes A, B, C with probabilities q_1, q_2, q_3 and use the indifference principle. Alice mixes her strategies because they fare equally well against Bob's mixed strategy, hence $2q_1 - q_3 = -2q_1 + q_2 + 3q_3 = q_1 + q_2 - 2q_3$ which together with $q_1 + q_2 + q_3 = 1$ give $q_1 = 5/16, q_2 = 1/2, q_3 = 3/16$. Bob mixes his strategies because they fare equally well against Alice's mixed strategy, hence $2p_1 - 2p_2 + p_3 = p_2 + p_3 = -p_1 + 3p_2 - 2p_3$ which together with $p_1 + p_2 + p_3 = 1$ give $p_1 = 9/16, p_2 = 3/8, p_3 = 1/16$.

2(ii)(a) Write

$$A = \begin{bmatrix} 2 & 0 & -1 \\ -2 & 1 & 3 \\ 1 & 1 & -2 \end{bmatrix}$$

We compute $[p_1 p_2 p_3]A = [7/16\ 7/16\ 7/16]$ and see that no matter what Bob plays, (p_1, p_2, p_3) guarantees Alice at least 7/16. We compute $A[q_1\ q_2\ q_3]^t = [7/16\ 7/16\ 7/16]^t$ and see that no matter what Alice plays, $(q_1\ q_2\ q_3)$ guarantees Bob to lose no more than 7/16. We deduce that the value of the game is 7/16 and that the strategies we found in (a) are optimal.

3(i)(a) Describe the game in tree form and perform backward induction.



If Klingons do burn their spaceships, they will have to fight at K_2, and so Romulans would prefer to abandon at R_1 and Klingons will receive 2 units of utility. If they don't burn their spaceships, when facing a decision at K_3 they will decide to run, and so Romulans will decide to stay at R_2, resulting in 0 units of utility for the Klingons. So the Klingon general will order the spaceships burned.

3(i)(b) Klingons' strategies amount to choosing action a at at K_1 and b at K_3 which we denote $[a, b]$. Romulans' strategies amount to choosing action c at R_1 and d at R_2 which we denote $[c, d]$. The game in strategic form is given by the following table, where Klingons are the row players and Romulans are the column players

	$[Abandon, Abandon]$	$[Abandon, Stay]$	$[Stay, Abandon]$	$[Stay, Stay]$
$[Burn, Run]$	2,1	2, 1	0, 0	0, 0
$[Burn, Fight]$	2,1	2, 1	0, 0	0, 0
$[DontBurn, Run]$	2,1	1, 2	2, 1	1, 2
$[DontBurn, Fight]$	2,1	0, 0	2, 1	0, 0

3(i)(c) It is easy to verify that ([Dont Burn, Fight], [Abandon, Abandon]) is a Nash equilibrium, but it is not subgame perfect. To see this consider the subgame starting at K_3: this strategy call for the Klingons to fight, which is certainly not a Nash equilibrium as they gain by running instead.

3(ii) Assume that there is no strategy which guarantees victory or a draw to the first player. Zermelo's Theorem implies that there exists a strategy for the second player which guarantees him victory.

The first player starts by picking any random square S and henceforth plays the game as if she were the second player and as if S were empty. If, however, this strategy calls for taking square S, Alice makes a random move. If, later in the game, the strategy calls for taking a square already owned by the first player, she makes any random move. The game ends either with a winning position in the second player's winning strategy book (with players interchanged), or with such a wining position plus an extra square for the first player, which is also a winning position.

This gives the first player a strategy which guarantees him victory and is a contradiction to our initial assumption.

4(i) Let s_1 be the strategy for player 1 in which, if player 2 ever deviated from A she plays II and she plays I as long as player 2 sticks to A. Let s_2 be the strategy for player 2 in which, if player 1 ever deviated from I he plays B and he plays A as long as player 1 sticks to I.

If (s_1, s_2) is played, the players get a payoff of 0 each.

If player 1 deviates from playing I by playing II at the k stage of the game and x_i $(i > k)$ thereafter, she gets the payoff

$$\sum_{i=0}^{k-1} 0 \times p^i + 4 \times p^k + \sum_{i=k+1}^{\infty} p^i u_1(x_i, B) \leq 4 \times p^k - \sum_{i=k+1}^{\infty} p^i = 4 \times p^k - \frac{p^{k+1}}{1-p}$$

so the first player wont deviate precisely when $4 \times p^k - \frac{p^{k+1}}{1-p} < 0$ which occurs when $p > 4/5$.

If player 2 deviates from playing A by playing B at the k stage of the game and y_i $(i > k)$ thereafter, he gets the payoff

$$\sum_{i=0}^{k-1} 0 \times p^i + 5 \times p^k + \sum_{i=k+1}^{\infty} p^i u_2(II, y_i) \leq 5 \times p^k - \sum_{i=k+1}^{\infty} p^i = 5 \times p^k - \frac{p^{k+1}}{1-p}$$

so the first player wont deviate precisely when $5 \times p^k - \frac{p^{k+1}}{1-p} < 0$ which occurs when $p > 5/6$.

We deduce that (s_1, s_2) is a Nash equilibrium precisely when $5/6 = \max\{4/5, 5/6\} < p < 1$.

4(ii)(a) Bob has two types: good mood which we denote B_g and bad mood which we denote B_b. Bob strategies consist of all function from the set $\{B_g, B_b\}$ to the set of actions { greet, dont greet}. Thus his four strategies are $[B_g \to$ greet$, B_b \to$ greet$]$, $[B_g \to$ ignore$, B_b \to$ greet$]$, $[B_g \to$ greet$, B_b \to$ ignore$]$, $[B_g \to$ ignore$, B_b \to$ ignore$]$ which we abbreviate [greet, greet], [ignore, greet], [greet, ignore], [ignore, ignore].

4(ii)(b) The expected payoffs of each of Alice's actions against each one of Bob's strategies are given as follows

	[greet, greet]	[ignore, greet]	[greet, ignore]	[ignore, ignore]
greet	$\underline{0}$	$\underline{-5}$	$\underline{0}$	-5
ignore	-9	$\underline{-5}$	-6	$\underline{-2}$

where best responses were underlined.

4(ii)(c) If Alice greets, Bob does best greeting her regardless of his type, and if she ignores him he does best by ignoring her regardless of his type.

4(ii)(d) We compile the following payoff table

	[greet, greet]	[ignore, greet]	[greet, ignore]	[ignore, ignore]
greet	$\underline{0}$, ($\underline{5}$,$\underline{-3}$)	$\underline{-5}$, (-10,$\underline{-3}$)	$\underline{0}$, ($\underline{5}$, -4)	-5, (-10, -4)
ignore	-9, (-5, -20)	$\underline{-5}$, ($\underline{-1}$, -20)	-6, (-5, $\underline{0}$)	$\underline{-2}$, ($\underline{-1}$, $\underline{0}$)

where best responses for Alice and both Bobs are underlined. We note Nash equilibria at (greet, [greet, greet]) and (ignore, [ignore, ignore]).