The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS — Autumn 2013-2014

Game Theory MOCK EXAM

2 hours and 30 minutes

Attempt all the questions. The allocation of marks is shown in brackets.

Please leave this exam paper on your desk
Do not remove it from the hall

Registration number from U-Card (9 digits)
to be completed by student

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1 (i) Alice and Bob are given £4 to divide among themselves and they agree to do so as follows. Both pick simultaneously a non-negative integer less or equal to 4 and

- if the integers add up to 4 or less, each gets the amount they named,
- if the integers are different and add up to more than 4, the person who chose the smaller number gets that amount and the other person gets the rest, and
- if the integers are equal and add up to more than 4, each gets £2.

(a) Describe this situation as a game in strategic form. ***(5 marks)***

(b) Find all strategies which are strictly dominated, and all strategies that are weakly dominated. ***(3 marks)***

(c) Find all pure-strategy Nash-equilibria of this game. ***(4 marks)***

(d) Show that the sequential elimination of weakly dominated strategies can lose Nash equilibria. ***(3 marks)***

(ii) Suppose that two firms produce similar but not identical products, and that the unit costs of these products are £10 for firm 1 and £20 for firm 2. The prices of each of these two products depend on the production profile (q_1, q_2) of both products: $p_1 = 100 - 6q_1 - 2q_2$ and $p_2 = 200 - 5q_1 - 7q_2$. Assume that each firm i controls its production profile q_i.

(a) Find each company's production which is the best response to the other company's production. ***(6 marks)***

(b) Find the production profile which is a Nash-equilibrium. ***(4 marks)***

2 (i) Consider a finite zero-sum game (S, T, u) and let Δ^R and Δ^C be the sets of mixed strategies of the row and column players, respectively.

(a) Show that

$$\max_{p \in \Delta^R} \min_{q \in \Delta^C} u(p, q) = \max_{p \in \Delta^R} \min_{t \in T} u(p, \widehat{t})$$

where $\widehat{t}$ is the strategy which plays t with probability 1.

(10 marks)

(b) Let V be the value of this game. Show that p^* is an optimal strategy for the row player if and only if $V = \min_{t \in T} u(p^*, \widehat{t})$. *(6 marks)*

(ii) Consider the following zero-sum game given in tabular form

	A	B	C
I	2	0	-1
II	-2	1	3
III	1	1	-2

(a) Find an optimal strategy pair (p, q) under the assumption that both strategies have all pure strategies in their support. *(5 marks)*

(b) Verify that the strategy pair (p, q) is indeed optimal, and find the value of the game. *(4 marks)*

3 (i) The Klingons (a belicose alien civilization) invade the planet Romulus, and the Romulans need to decide whether to abandon their planet or to stay put. If they stay and the Klingons fight, both get a payoff of 0, whereas if the Klingons run away, Romulans get 2 and Kilngons get 1. If the Romulans run away, they get 1 and the Klingons get 2.

Immediately after landing on Romulus, the Klingon commander needs to decide whether to destroy her spaceships (and thus eliminating the option to run away if Romulans decide to stay put).

(a) Assuming everyone is rational and well informed, what should the Klingon commander do? Explain you reasoning in detail.

(6 marks)

(b) Describe this situation in detail as a two-player game in strategic form. *(6 marks)*

(c) Find a pure strategy Nash equilibrium of this game which is not subgame perfect. *(4 marks)*

(ii) *Gomoku* is a game played by two players on a $n \times n$ board. Players alternate putting their pieces on the board, and the first person to have five consecutive pieces on a row, column or diagonal wins the game. If all squares are filled with pieces, and no player won, the game is drawn.

Show that there exists a strategy that guarantees victory or a draw to the first player. *(9 marks)*

4 (i) Consider the following two-player game given in strategic form in the following table

	A	B
I	(0, 0)	(-2, 5)
II	(4, -1)	(-1, -1)

and consider the indefinitely repeated game $G(p)$ consisting of playing G repeatedly, and after each such subgame, stopping with probability $1 - p$.

Consider the row players's strategy s_1 consisting of playing I as long as the other player has always played A, and playing II otherwise. Consider also the column players's strategy s_2 consisting of playing A as long as the other player has always played I, and playing B otherwise.

For which values of p is (s_1, s_2) a Nash equilibrium? ***(9 marks)***

(ii) Alice sees her neighbour Bob who is either in a good mood or in a bad mood, with equal probability. Both Alice and Bob must decide simultaneously and independently to either greet or ignore the other. The payoffs of the various actions in both scenarios are as follows

Bob is in a good mood

	Greet	Ignore
greet	5, 5	-5, -10
ignore	-10, -5	-2, -1

Bob is in a bad mood

	Greet	Ignore
greet	-5, -3	-5, -4
ignore	-8, -20	-2, 0

We model this situation as a Bayesian game.

(a) Describe the set of all strategies for Bob. ***(2 marks)***

(b) Describe expected payoffs of Alice's actions against each of Bob's strategies and find her best responses for each of Bob's strategies. ***(5 marks)***

(c) Describe Bob's best responses against each of Alice's possible actions. ***(5 marks)***

(d) Find the Bayes-Nash equilibria of this game ***(4 marks)***

End of Question Paper