Game Theory

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Administratrivia

Office Hours: Monday 11am and Wednesday 2pm (room J16)
Exam: 2.5 hours, 4 questions out of 4.
Homework: Four assignments.

What is a Game

A game consists of

(a) a set of players, each having

(b) a set of actions or strategies; a choice of strategies by all players determines

(c) outcomes (or payoffs) for each player.

Examples:

- Rock-paper-scissors
- Sealed-bid auctions (many players, partial information)
- M.A.D. (Mutually Assured Destruction) (scary game from my childhood)
- Duopolies, Oligopolies,
- Chess (sequential)
- Chomp!
- backgammon (partial information, potentially infinite)

Example: Prisoners' Dilemma

Alice and Bob are arrested for murder and theft, but while there is enough evidence to convict them of theft, there is not enough evidence to convict them of murder, unless one or both prisoners confess. Alice and Bob are both told:

> *"If you confess and your partner does not, you go free and your partner will be jailed 20 years. If neither of you confess, you will go to jail for a year. If both of you confess, you will both go to jail for 10 years."*

What's a prisoner to do?

		Bob	
		Confess	Don't Confess
Alice	Confess	Alice: 10 years, Bob: 10 years	Alice: free, Bob: 20 years
	Don't Confess	Alice: 20 years, Bob: free	Alice: 1 year, Bob: 1 year

Preferences:

going free is best,

1 year in jail preferable to 10 years,

10 years in jail preferable to 20 years,

(1 year in jail preferable to 20 years.)

Lets play!

		Bob	
		Confess	Don't Confess
Alice	Confess	Alice: 10 years, Bob: 10 years	Alice: free, Bob: 20 years
	Don't Confess	Alice: 20 years, Bob: free	Alice: 1 year, Bob: 1 year

Take out a piece of paper

Now you are in jail: Pair up with the person sitting next to you (your partner in crime), chat and negotiate, meanwhile write in a piece of paper "Confess" or "Not Confess" **without your partner seeing what you write** and when I say *Now!* reveal your decision to your partner.

Optimal strategy: *Confess*.
Don't Confess is dominated by *Confess*: it always does worse!

Lets play TWICE!

		Bob	
		Confess	Don't Confess
Alice	Confess	Alice: 10 years, Bob: 10 years	Alice: free, Bob: 20 years
	Don't Confess	Alice: 20 years, Bob: free	Alice: 1 year, Bob: 1 year

Take out a piece of paper

You are in jail again in the same situation as before with the same partner in crime. BUT now you know that you will face exactly the same situation twice in your life.

Chat and negotiate with your partner in crime, meanwhile write in a piece of paper your decision ("Confess" or "Not Confess") for your first arrest **without your partner seeing what you write** and when I say *Now!* reveal your decision to your partner.

Write in a piece of paper your decision for your second arrest **without your partner seeing what you write** and when I say *Now!* reveal your decision to your partner.

Lets play INDEFINITELY!

		Bob	
		Confess	Don't Confess
Alice	Confess	Alice: 10 years, Bob: 10 years	Alice: free, Bob: 20 years
	Don't Confess	Alice: 20 years, Bob: free	Alice: 1 year, Bob: 1 year

You are in jail again in the same situation as before with the same partner in crime. BUT now you know that you may face exactly the same situation many times in your life: each time you get out of jail a coin is tossed and if it lands on "Heads", you'll find yourself in the same situation in 10 years, otherwise this nightmare stops. Chat and negotiate with your partner in crime, and come up with a plan.

Utility and preferences

We want to compare attractiveness of outcomes of games.
Utility: a real number which is the measure of attractiveness of an outcome.
High utility preferable to low utility.
Game players aim to maximize utility (or expected utility later in the course).

Example

Alice's utilities of 0, 1, 10 and 20 years in jail are 100, 3, 0, -1.
Bob's utilities of 0, 1, 10 and 20 years in jail are 10, -5, -10, -20.

	Confess	Don't Confess
Confess	(0,-10)	(100,-20)
Don't Confess	(-1,10)	(3,-5)

If we change, say, 100, 3, 0, -1, with any $a > b > c > d$ the analysis doesn't change.