MAS348 Game Theory
Problem Sheet #1

Problems marked with "#" are more difficult, and optional

1. Consider the following game in strategic form

	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	3,0

   Find all strictly dominated strategies and all weakly dominated strategies.

2. Consider the following game in strategic form

	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	7, 5

   Find the best responses to each strategy and find all (pure strategy) Nash equilibria.

3. Here we reanalyze the Mean Voter Theorem in a continuous setting: Alice and Bob run for elections on one issue which is is modeled by one number. For each $0 \leq x \leq 1$ adopting position x will mean adopting a position which equals or exceeds the position of proportion of x the population. Consider the elections as a 2-player game with two outcomes: *Alice wins* and *Alice loses* (we could consider also the possibility of a draw, but we ignore it.) Show that

   (a) if $1/2 \leq x_1 < x_2 \leq 1$, adopting position x_1 weakly dominates adopting position x_2,

   (b) if $0 \leq x_1 < x_2 \leq 1/2$, adopting position x_2 weakly dominates adopting position x_1, and, therefore

   (c) the only non-weakly dominated position is $x = 1/2$.

4. Alice and Bob are bargaining how to split £100. They simultaneously announce how much they would like to receive, say a and b ($0 \leq a, b \leq 100$). If $a + b \leq 100$, Alice receives $£a$ and Bob $£b$; if $a + b > 100$ they both get nothing. Are there any dominated or weakly dominated strategies in this game? What are the Nash equilibria of this game? What would be a good strategy for this game? Would you use the same strategy against Mother Teresa and against Vladimir Putin?

5. # As in Problem 4 Alice and Bob are splitting £100. However, now Alice moves first and announces her share $0 \leq a < 100$. Describe this game in strategic form, specifically describe the set of strategies available to Bob. Find all dominated strategies and Bob's best response to Alice's a.

6. Three voters vote over two candidates (A and B), and each voter has two pure strategies: vote for A and vote for B. When A wins, voter x gets a payoff of 1, and y and z get payoffs of 0; when B wins, x gets 0 and y and z get 1. Thus, x prefers A, and y and z prefer B. The candidate getting 2 or more votes is the winner (majority rule).

   Find all weakly dominant strategies and all pure strategy Nash equilibria.

7. Alice and Bob are bidding for a government contract. Whoever gets the contract will make a profit of £1,000,000 while the loser gets nothing. Alice and Bob spend $\pounds a$ and $\pounds b$, respectively, preparing their bids, and consequently, their probability of winning the contract are $a/(a+b)$ and $b/(a+b)$ respectively. Find the expenditures and expected profits at a Nash equilibrium and compare the expected profits with what they could obtain if they had colluded. Visit the *Competition and Markets Authority* and find out about government attempts to stop collusion in private tenders.

8. A group of n manufacturers can produce as many widgets as they wish at no cost; if the combined production is S, the price per widget will be e^{-S}. Show that the dominating strategy for a profit-maximizing manufacture is to produce one unit. Verify that if all manufacturers agreed to manufacture only $1/n$ units, they would all be better off, and that this would produce the largest total profit.

9. A group of people choose a real number between 0 and 100 and the winner is the one who chooses a number closest to twice the average. What are the Nash equilibria?

10. Consider a Cournot duopoly where the unit costs of the two firms are different, say c_1 and c_2, respectively. Again, the firms choose their production profile (q_1, q_2) resulting in a price $p(q_1, q_2) = a - b(q_1 + q_2)$. Find the best response functions of both firms, the Nash equilibrium production profile (q_1^*, q_2^*) and the profits at that equilibrium.

11. Suppose that two firms produce similar but not identical products, and that the unit costs of these products are c_1 and c_2. The prices of each of these two products depend on the production profile (q_1, q_2) of both products: $p_1 = a - b(2q_1 + q_2)$ and $p_2 = a - b(q_1 + 2q_2)$. Assume that each firm i controls its production profile q_i. Generalize the Cournot duopoly model to this setup.

12. # Recall the setup of the Cournot duopoly model in which two firms decide on production levels q_1 and q_2 of products with production costs of c per unit and decreasing price function $p(q_1 + q_2)$. Assume further that the price function is differentiable. Consider the profit functions $f_1(q_1, q_2) = (p(q_1 + q_2) - c)\, q_1$ and $f_2(q_1, q_2) = (p(q_1 + q_2) - c)\, q_2$. Define the *isoprofit curves* of firms 1 and 2 to be the sets of points on the $q_1 q_2$-plane where f_1 and f_2 are constant. Show that if the firms are allowed to collude, they will choose production levels corresponding to points (q_1^*, q_2^*) on which the two firms' isoprofit curves are tangent.

(Hint: we expect this production profile to be *Pareto efficient*, i.e., to be such that there is no other production profile which benefits one firm without harming the other.