# MAS348 Game Theory
# Problem Sheet #1

*Problems marked with "#" are more difficult, and optional*

1. Consider the following game in strategic form

| | x | y | z |
|---|---|---|---|
| a | 1,2 | 2,2 | 5,1 |
| b | 4,1 | 3,5 | 3,3 |
| c | 5,2 | 4,4 | 7,0 |
| d | 2,3 | 0,4 | 3,0 |

   Find all strictly dominated strategies and all weakly dominated strategies.

2. Consider the following game in strategic form

| | x | y | z |
|---|---|---|---|
| a | 1,2 | 2,2 | 5,1 |
| b | 4,1 | 3,5 | 3,3 |
| c | 5,2 | 4,4 | 7,0 |
| d | 2,3 | 0,4 | 7, 5 |

   Find the best responses to each strategy and find all (pure strategy) Nash equilibria.

3. Here we reanalyze the Mean Voter Theorem in a continuous setting: Alice and Bob run for elections on one issue which is is modeled by one number. For each $0 \leq x \leq 1$ adopting position $x$ will mean adopting a position which equals or exceeds the position of proportion of $x$ the population. Consider the elections as a 2-player game with three outcomes: *Alice wins, Bob loses*, *Alice loses, Bob wins* and *tie*. Show that

   (a) if $1/2 \leq x_1 < x_2 \leq 1$, adopting position $x_1$ weakly dominates adopting position $x_2$,

   (b) if $0 \leq x_1 < x_2 \leq 1/2$, adopting position $x_2$ weakly dominates adopting position $x_1$, and, therefore

   (c) the only non-weakly dominated position is $x = 1/2$.

4. Alice and Bob are bargaining how to split £100. They simultaneously announce how much they would like to receive, say $a$ and $b$ ($0 \leq a, b \leq 100$). If $a + b \leq 100$, Alice receives $\pounds a$ and Bob $\pounds b$; if $a + b > 100$ they both get nothing. Are there any dominated or weakly dominated strategies in this game? What are the Nash equilibria of this game? What would be a good strategy for this game? Would you use the same strategy against Mother Teresa and against Vladimir Putin?

5. # As in Problem 4 Alice and Bob are splitting £100. However, now Alice moves first and announces her share $0 \leq a < 100$. Describe this game in strategic form, specifically describe the set of strategies available to Bob. Find all dominated strategies and Bob's best response to Alice's $a$.

6. Three voters vote over two candidates (A and B), and each voter has two pure strategies: vote for A and vote for B. When A wins, voter x gets a payoff of 1, and y and z get payoffs of 0; when B wins, x gets 0 and y and z get 1. Thus, x prefers A, and y and z prefer B. The candidate getting 2 or more votes is the winner (majority rule).

   Find all weakly dominant strategies and all pure strategy Nash equilibria.

7. Alice and Bob are bidding for a government contract. Whoever gets the contract will make a profit of £1,000,000 while the loser gets nothing. Alice and Bob spend $£a$ and $£b$, respectively, preparing their bids, and consequently, their probability of winning the contract are $a/(a+b)$ and $b/(a+b)$ respectively. Find the expenditures and expected profits at a Nash equilibrium and compare the expected profits with what they could obtain if they had colluded. Visit the *Competition and Markets Authority* and find out about government attempts to stop collusion in private tenders.

8. A group of $n$ manufacturers can produce as many widgets as they wish at no cost; if the combined production is $S$, the price per widget will be $e^{-S}$. Show that the dominating strategy for a profit-maximizing manufacture is to produce one unit. Verify that if all manufacturers agreed to manufacture only $1/n$ units, they would all be better off, and that this would produce the largest total profit.

9. A group of people choose a real number between 0 and 100 and the winner is the one who chooses a number closest to twice the average. What are the Nash equilibria?

10. Consider a Cournot duopoly where the unit costs of the two firms are different, say $c_1$ and $c_2$, respectively. Again, the firms choose their production profile $(q_1, q_2)$ resulting in a price $p(q_1, q_2) = a - b(q_1 + q_2)$. Find the best response functions of both firms, the Nash equilibrium production profile $(q_1^*, q_2^*)$ and the profits at that equilibrium.

11. Suppose that two firms produce similar but not identical products, and that the unit costs of these products are $c_1$ and $c_2$. The prices of each of these two products depend on the production profile $(q_1, q_2)$ of both products: $p_1 = a - b(2q_1 + q_2)$ and $p_2 = a - b(q_1 + 2q_2)$. Assume that each firm $i$ controls its production profile $q_i$. Generalize the Cournot duopoly model to this setup.

12. $^{\#}$ Recall the setup of the Cournot duopoly model in which two firms decide on production levels $q_1$ and $q_2$ of products with production costs of $c$ per unit and decreasing price function $p(q_1 + q_2)$. Assume further that the price function is differentiable. Consider the profit functions $f_1(q_1, q_2) = (p(q_1 + q_2) - c)\, q_1$ and $f_2(q_1, q_2) = (p(q_1 + q_2) - c)\, q_2$. Define the *isoprofit curves* of firms 1 and 2 to be the sets of points on the $q_1q_2$-plane where $f_1$ and $f_2$ are constant. Show that if the firms are allowed to collude, they will choose production levels corresponding to points $(q_1^*, q_2^*)$ on which the two firms' isoprofit curves are tangent. (Hint: we expect this production profile to be *Pareto efficient*, i.e., to be such that there is no other production profile which benefits one firm without harming the other.