MAS348 Game Theory
Problem Sheet #3

1. Consider a game played on a $2n \times 2n$ board in which white and black alternate in putting a piece in a square. The first person to have $2n$ pieces of their colour in a row or in a column loses. Find a strategy that guarantees the player who doesn't start (black) a draw. Hint: think of a strategy-stealing argument in which black mimics white's moves.

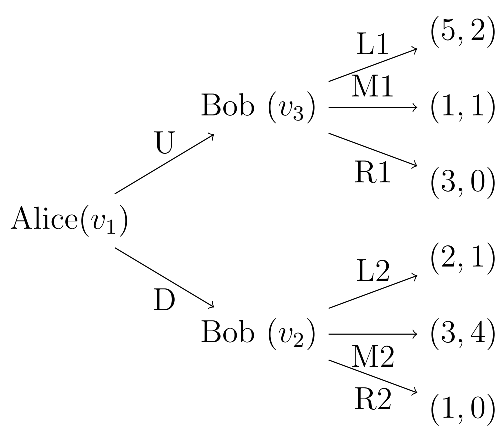
2. (*Bachet's Game.*) There are n tokens on the table. Alice and Bob take turns removing any number of tokens between 1 and k from the table, and Alice goes first. The winner is the one to take the last token. For which values of n and k does Alice have a strategy which guarantees her victory over Bob?

3. Five pirates named Alice, Bob, Charles, David and Eve have just seized a hundred gold coins, and now its time to share the loot as follows. Whoever's name is first in alphabetical order proposes an division of the one hundred coins to the remaining pirates. If a strict majority accepts the proposal, then the coins are allocated thus and the allocation process ends. Otherwise, if the proposal fails, then the proposer gets thrown overboard and the game is repeated with the remaining pirates. What should Alice propose?

4. Alice plays tic-tac-toe against Bob and she moves first. Prove that there exists a strategy which guarantees Alice at least a draw.

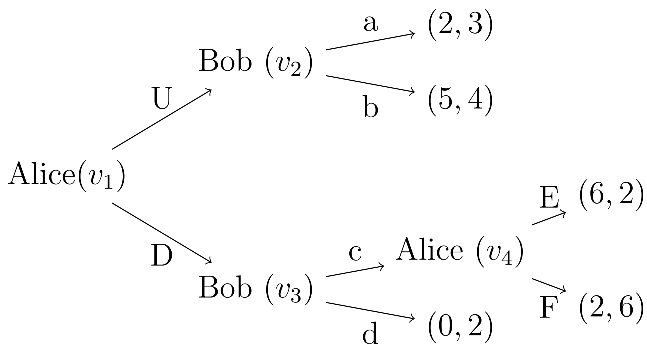
5. Describe a winning strategy for the first player playing *Chomp!* with a square chocolate bar.

6. Consider the following sequential game G whose tree T is as follows:



   (a) Solve this game.
   (b) Describe this game in normal form and find its Nash equilibria.
   (c) Which of the Nash equilibria in (b) are subgame-perfect?

7. Consider the following sequential game G whose tree T is as follows:



(a) Can this game be "solved" by using backward induction?

(b) Describe this game in normal form and find its pure strategy Nash equilibria.

(c) Which of the Nash equilibria in (b) are subgame-perfect?

8. Alice and Bob play the game of *chicken* as described in Problem Sheet 2, but with a twist. Alice's steering wheel can be detached, and so she has the choice of throwing her steering wheel out of the window before she and Bob decide whether to swerve or not. Assume that if she throws her steering wheel, this would be observed by Bob. Model this game as a sequential game with imperfect information and find all its subgame perfect Nash equilibria. Describe one Nash equilibrium which is not subgame perfect. What should Alice do?