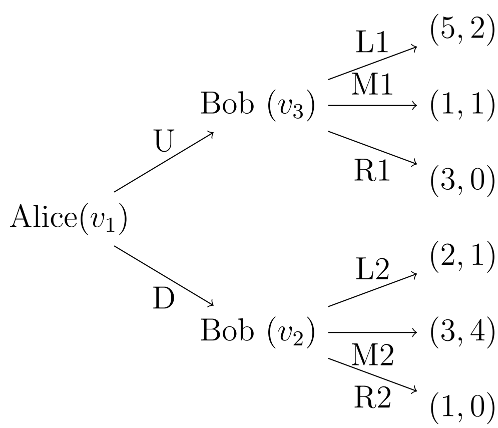
MAS348 Game Theory
Problem Sheet #3

1. Consider a game played on a $2n \times 2n$ board in which white and black alternate in putting a piece in a square. The first person to have $2n$ pieces of their colour in a row or in a column loses. Find a strategy that guarantees the player who doesn't start (black) a draw. Hint: think of a strategy-stealing argument in which black mimics white's moves.

2. (*Bachet's Game.*) There are n tokens on the table. Alice and Bob take turns removing any number of tokens between 1 and k from the table, and Alice goes first. The winner is the one to take the last token. For which values of n and k does Alice have a strategy which guarantees her victory over Bob?

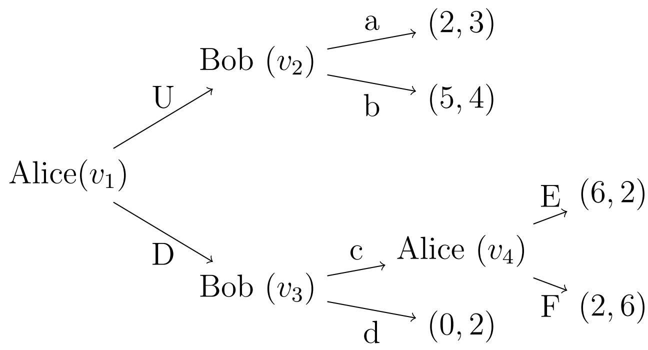
3. Alice plays tic-tac-toe against Bob and she moves first. Prove that there exists a strategy which guarantees Alice at least a draw.

4. Consider the following sequential game G whose tree T is as follows:



   (a) Solve this game.
   (b) Describe this game in normal form and find its Nash equilibria.
   (c) Which of the Nash equilibria in (b) are subgame-perfect?

5. Consider the following sequential game G whose tree T is as follows:



(a) Can this game be "solved" by using backward induction?

(b) Describe this game in normal form and find its pure strategy Nash equilibria.

(c) Which of the Nash equilibria in (b) are subgame-perfect?