

# Forward and Futures Contracts

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James Mackintosh looks at this month's short squeezes

## **Short selling bets against Tiffany at two-year high Financial Times. Monday, March 11, 2013**

Bets against shares in Tiffany & Co touched a two-year high this month as the fine jeweller battles higher inventory costs, falling sales and weakening customer loyalty.

Market short sellers - investors who sell stock they do not own, hoping to profit by buying it back at a lower price later- increased their positions dramatically ahead of the company's fourth-quarter earnings, due on March 22. According to the latest data compiled by Markit and Bloomberg, short interest in the New York retailer rose 10 per cent over the past week.

Tiffany cut its annual profit forecast three times last year following lacklustre sales in its core US and European markets and a legal battle with Swiss watch group Swatch. Tiffany is also suffering from reduced interest in its fashion jewellery - pieces priced at close to \$500.

Just over 13% of Tiffany's 126.8m outstanding shares on the New York Stock Exchange were held by short sellers on Friday.

Its shares rose 63 cents to \$70.25 on Friday, close to their 52-week high of \$70.71.

## Forward and futures contracts; short selling

We will describe simplest derivatives: forward contracts and futures contracts.

We will apply no-arbitrage arguments to describe the “correct” forward and futures prices.

**Assumption:** any tradable asset can be bought in positive and in negative quantities.

Buying a negative quantity of an asset, called short selling, amounts to selling something you don't own!

## How does short-selling work?

You short sell an asset through a broker, the broker borrows the asset from another client and sells it, broker gives you the proceeds.

To “close” a short position, buy asset and return it.

A profit is made if the sale price was higher than the purchase price, i.e., if the asset's value depreciated.

While the short position exists one has to maintain a margin, an amount of money which is a percentage of the value of the short position to protect against adverse price movements.

The margin is balanced daily.

## 10 minute Quiz!

Prove the following:

**Theorem 2.0:** Let  $p_1$  and  $p_2$  be the current prices of two assets, and suppose that we know with certainty that their prices  $q_1$  and  $q_2$  at some point in the future satisfy  $q_1 \leq q_2$ . Then  $p_1 \leq p_2$ .

## Further assumptions:

There are no transaction costs.

There are no taxes

Traders can borrow and lend money at the same risk-free rate.

There are no arbitrage opportunities.

## Forward contracts.

A forward contract is an agreement to deliver an asset at an agreed price  $F$ , the forward price, and at an agreed date, the maturity date or delivery date.

There is an asymmetry in the parties to a forward contract:

The party receiving the asset has a long position and the party delivering the asset has a short position.

(In finance, “short” often means “sell” and “long” often stands for “buy”.)

*Entering a forward agreement involves no investment!*



## Assets providing income

Some assets provide income, e.g., dividend yielding shares in a company.

A bar of gold is example of an asset which pays no income.

## What is a correct forward price?

Consider first a forward contract on an asset which provides no income.

**Theorem 2.1:** Let  $S$  be the spot price of the asset, i.e., the price of the asset at  $t = 0$ .

The forward price is  $F = Se^{rT}$  where  $T$  is the time to maturity and  $r = Y(T)$ , the  $T$ -year spot interest rate.

**Proof:** If  $F > Se^{rT}$  there is the following arbitrage strategy: take a short position in the forward contract, borrow  $\pounds S$  for  $T$  years at spot interest rate of  $r$ , buy the asset for  $\pounds S$ , wait  $T$  years, deliver the asset and collect  $\pounds F$ , use  $\pounds Se^{rT}$  to repay the loan, pocket the difference  $F - Se^{rT} > 0$ .

If  $F < Se^{rT}$  there is the following (in a sense opposite) arbitrage strategy:

take a long position in the forward contract, short sell the asset for  $\pounds S$ , deposit  $\pounds S$  for  $T$  years at spot interest rate of  $r$ , wait  $T$  years, withdraw  $\pounds Se^{rT}$  from your deposit, have the asset delivered for  $\pounds F$ , use it to close the short position on the asset, pocket the difference  $Se^{rT} - F > 0$ .  $\square$

## Forward prices of assets generating income

We now find the correct forward prices when the underlying asset provides income.

### **Theorem 2.2:**

Consider a forward contract on an asset that provides income, with maturity date in  $T$  years. Let  $S$  be the spot price of the asset. Let  $I$  be the present value of the income generated by the asset until time  $T$ . Then  $F = (S - I)e^{rT}$  where  $r = Y(T)$ , the  $T$ -year spot interest rate.

**Proof:** Suppose that the asset produces payments at times  $0 < t_1 < \dots < t_n < T$  whose present values are  $I_1, \dots, I_n$ . We write  $r_1 = Y(t_1), \dots, r_n = Y(t_n)$ .

If  $F > (S - I)e^{rT}$  there is the following arbitrage strategy:

Take a short position in the forward contract. Borrow  $\mathcal{L}(S - I)$  for  $T$  years at spot interest rate of  $r$  for every  $1 \leq k \leq n$  borrow  $I_k$  for  $t_k$  years at a spot rate of  $r_k$

Buy the asset for  $\mathcal{L}S$

Wait  $T$  years: meanwhile use the payments from the asset to repay the corresponding loan. At the end of this period the outstanding debt is  $\mathcal{L}(S - I)e^{rT}$ .

Deliver the asset and collect  $\mathcal{L}F$ ,

Use  $\mathcal{L}(S - I)e^{rT}$  to repay the loan,

Pocket the difference  $F - (S - I)e^{rT} > 0$ .

If  $F < (S - I)e^{rT}$  there is the following arbitrage strategy:

Take a long position in the forward contract.

Short sell the asset for  $\pounds S$ .

Deposit  $\pounds(S - I)$  for  $T$  years at spot interest rate of  $r$ ; for every  $1 \leq k \leq n$  deposit  $I_k$  for  $t_k$  years at a spot rate of  $r_k$ .

Wait  $T$  years: meanwhile make the payments due from the asset while withdrawing the corresponding deposits.

At the end of this period the deposit balance is  $\pounds(S - I)e^{rT}$ .

Withdraw  $\pounds(S - I)e^{rT}$  from your deposit.

Have the asset delivered for  $\pounds F$ .

Use it to close the short position on the asset

Pocket the difference  $(S - I)e^{rT} - F > 0$ .  $\square$

## Foreign exchange forward contracts

Consider a forward contract on one unit of foreign currency whose spot rate is  $S$  i.e., it costs  $S$  units of domestic currency to purchase one unit of foreign currency.

Let the maturity date be  $T$  years in the future, and let the yield curve in the foreign currency be  $Y_f(t)$ .

Let  $F$  be the forward rate, i.e., the party with the short position in the forward contract will deliver in  $T$  years one unit of foreign currency and receive a payment of  $F$  units of domestic currency.

Again we ask, what should  $F$  be?

Write  $r = Y(T)$  and  $r_f = Y_f(T)$ ;

**Theorem 2.3:**  $F = Se^{(r-r_f)T}$

**Proof:** Consider the following two portfolios:

**Portfolio A:** a long position in the forward contract and  $Fe^{-rT}$  units of domestic currency earning interest rate of  $r$  for  $T$  years.

**Portfolio B:**  $e^{-r_f T}$  units of foreign currency earning interest rate  $r_f$ .

At time  $T$  portfolio A will generate  $F$  units of domestic currency which can be used to pay for the one unit of foreign currency obtained with the forward contract.

At time  $T$  portfolio B will also be worth one unit of foreign currency.

Our quiz implies that both portfolios must have the same value at any time  $t$  with  $0 \leq t \leq T$ .

If we consider  $t = 0$ , this gives us  $Fe^{-rT} = Se^{-r_f T}$ .  $\square$



## 10 minute Quiz! (2003-4 exam)

Suppose that the 1-year risk free interest rates for British Pounds (£) and US Dollars (\$) are 4% and 2%, respectively. Suppose further that the spot £/\$ exchange rate is now 0.54 (i.e., \$1 costs £0.54).

(a) What is the 1-year £/\$ forward exchange rate? (1 mark)

(b) Someone is willing to buy/sell in one year British pounds at a £/\$ exchange rate of 0.54. Describe in detail an arbitrage opportunity available to you. (10 marks)

## Futures contracts

Forward contracts are private agreements and not traded.

A futures contract is similar to a forward contract: it is also an agreement to deliver an asset at an agreed price  $F$ , the futures price, and at an agreed date, the maturity date.

Again, one can have a short position (a commitment to deliver the asset) or a long position (a commitment to buy the asset.)

Futures contracts are traded through brokers in specialised exchanges such as London's LIFFE and Chicago Board of Trade (CBOT).

These exchanges regulate both trading and delivery of assets.  
(Oil futures prices from the NY mercantile exchange.)

The underlying assets of futures contracts are standardized, e.g.,  
CME's wheat futures contract specification.

One closes out a position in a futures contract by entering an identical contract but with the opposite position, e.g., to close out a long position in a March 2014 Cocoa futures, you take a short position on a March 2014 Cocoa futures.

Futures *are settled daily*: e.g., assume you took a short position in a May 2014 Brent crude futures, with futures price \$24.10, i.e., you agreed to provide 1,000 US barrels of petroleum of a certain quality for \$24.10 per barrel.

## “Marking to market”

If at the end of the day the futures price for May 2014 Brent crude is \$24.20 then you lose \$0.10 per barrel, because now your barrel of oil can be delivered for \$0.10 more than what you will get.

You pay your broker  $\$1,000 \cdot 0.10 = \$100$  and your futures price is changed to \$24.20.

If the next day sees a drop of \$0.20 of the May 2014 Brent crude your broker will credit you \$200 and your new futures price is changed to \$24.

The daily settlement of futures is called marking to market.

# The Futures-Forwards Equivalence Principle

The marking to market of futures contracts invalidates the no-arbitrage arguments used to produce forward prices.

The reason for this is that the interest paid or obtained from the daily cash-flows generated by the futures contracts are *stochastic*, i.e., they are not known in advance.

However, one has the following.

**Theorem (The Futures-Forwards Equivalence Principle)** If interest rates are deterministic then forward price and futures price coincide.

**Proof:** Read chapter 3 of John Hull's *Options, Futures and other derivatives*.

The End