

MAS362/MAS462
Financial Mathematics

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Organizational matters

A summary of the material is available on the course webpage, please read it in advance of the lectures.

Printable versions of all slides will be available on the course webpage.

Feedback sessions/office hours:

Mondays at 11am and Wednesdays at 3pm (room J16)

If you need a special appointment, e-mail me at:

`M.Katzman@sheffield.ac.uk`

Homework

You will be assigned homework almost every week, starting from week 2.

I will collect and mark a subset of the assigned homework.

Hand in your work *before* the beginning of lecture. You can also put your work in my mailbox (common room, floor 1). I will post solutions— it is your responsibility to make sure you understand these.

Some feedback will be given in the marking, some in class, the rest during our twice-weekly feedback sessions.

Frequently unasked questions

The lecture notes are available in advance– do I really need to attend lectures?

Yes!

Do I really need do the homework? Can't I just learn this stuff before the exam?

If you don't practice the language of finance, you won't know even what the exam is asking.

If I do well in the course, will I get a fat cat job in the City, get ridiculously big bonuses and wreck the global economy?

Not right away– you'll need further practical training, but this course will give you the theoretical foundation for further study.

If I do very well in the course, could I work for the Bank of England or the FCA to prevent the fat cats above from wrecking our economy?

I hope you do! (with additional training)

Web page, textbooks, exam

The course web page is at

<http://www.shef.ac.uk/katzman/MAS362/MAS362.htm>.

You will find there useful information related to the course. Check the course web page periodically.

Recommended Books:

P. Wilmott, “ Paul Wilmott on quantitative finance”, Wiley (2007)

J. C. Hull, “Options, futures and other derivatives”, Prentice Hall (1997)

W. Sharpe, “Portfolio theory and capital markets”, McGraw-Hill (2000)

Assessment: One formal 2.5 hour exam.

Format: 4 out of 4 questions.

(**Closed** book exam.)

Etiquette

This is a large class and to prevent descent into noisy chaos we must follow the following firm rules.

- ▶ **Do not** arrive late to class. If you must enter the class late, please do so quietly and choose a seat near the door. If you are a chronic late comer, please discuss your reason with me.
- ▶ **Turn off** phones, beepers and any other noise making machines.
- ▶ **Do not** be shy– **do** ask questions!
- ▶ **Do not** talk during the lecture unless you are contributing to the discussion.

And now for our feature
presentation...

What is MAS362/MAS462 all about?

Trading post: someone's betting big on wobbly markets

The Vix index

I will teach you mathematics used in finance.

You will learn interesting *mathematical ideas*, and you will learn some basic facts about an important component of modern life: *finance*.

What is MAS362/MAS462 exactly about?

Specifically, we answer two questions:

1. What is the “correct price” of financial assets?
2. What are optimal investment strategies?

The first question will lead us to the *Black-Scholes pricing formula*, which earned Robert C. Merton and Myron S. Scholes the 1997 Nobel prize in economics.

William F. Sharpe and Harry M. Markowitz received the 1990 Nobel prize in economics for answering the second question.

“Correct price”? What's that?

What does “correct price” mean?

Let's start by understanding what an *incorrect price* might be with a very simple example.

Suppose that a barrel of oil trades at £100 (i.e., one can buy it for £100 and one can sell it for £100).

I want to start trading in oil, too. Would £120 be a correct price? No! Other traders will buy oil in large numbers for £100, sell it to me for £120. Everyone except me will become rich, I end up with a pile of barrels of oil I can't sell.

Would £90 be a correct price?

“Correct price”? What's that?

A more subtle example

Consider three merchants who are willing to buy and sell bags containing apples and oranges (all of identical size and quality) as follows:

	Bag content	Price
Merchant I	3 apples, 2 oranges	£5
Merchant II	2 apples, 3 oranges	£6
Merchant III	4 apples, 3 oranges	£8

Are these prices “correct”?

No!

Lets get rich:

- (a) borrow £36 for a short while (with negligible interest);
- (b) buy 6 bags from merchant I and buy 1 bag from merchant II for a total cost of £36;
- (c) rearrange the fruit in five bags of 4 apples and 3 oranges each;
- (d) sell the five bags to merchant III for £40;
- (e) return the £36 loan and pocket a £4 profit.

Repeat this process until you are very rich.

Lots and lots of people would be buying from merchants I and II and selling to merchant III.

When this happens the prices of bags I and II will rise and those of bag III will fall, and this process will continue until there are no more easy profits to be made.

The making of a certain profit with no investment is called arbitrage. Correct pricing = *the unique price which does not introduce arbitrage opportunities*.

The correct price of bag III would be $\pounds 36/5$ as that is the unique price for which the strategy above (or its opposite) does not produce a profit.

Arbitrage opportunities = “free lunches”

There is no such thing as a free lunch!

Notice

- (a) we didn't assume an intrinsic or objective price of apples and oranges,
- (b) correct pricing is an equilibrium price,
- (c) we expose incorrect pricing by exhibiting profit-making trading strategies which change prices.

Portfolio Theory

Are there investment strategies which are “better” than others?

But what do we mean by “better”?

We are greedy, i.e., we want high returns.

But average high returns are risky!

Suppose you are offered to make a bet

on the outcome of tossing a fair coin:

heads– you lose everything you own,

including your shirt and are forced to live on the street;

tails– you double your wealth.

Bets, anyone?

Expected gain = 0, but most people would not bet.

People are not only greedy– they also have an aversion towards risk.

Even if a win triples or quadruples their wealth, most people would not place a bet.

If we used a coin which produces tails with 99% probability *some* people might choose to place a bet.

Now the expected gain may tempt some, but not others.

There is an interplay between expected returns and risk.

A “good” way to invest is a strategy that compensates for its risk with an appropriately high expected return, depending on your risk-tolerance.

The last part of the course will make this statement precise: we will see that these optimal investments exist and consist of portfolios of risk-free deposits/bonds and market-index trackers.