

## MAS362/MAS462/MAS6053 Financial Mathematics Problem Sheet 1

1. Consider a £100,000 mortgage paying a fixed annual rate of 5% which is compounded monthly. What are the monthly repayments if the mortgage is to be repaid in 25 years?
2. You want to deposit your savings for a year and you have the choice of three accounts. The first pays 5% interest per annum, compounded yearly. The second pays 4.8% interest per annum, compounded twice a year. The third pays 4.9% interest per annum, compounded continuously. Which one should you choose?
3. Consider any traded asset whose value  $v_T$  in  $T$  years is known with certainty. Prove that the current value of the asset must be  $v_T e^{-rT}$  where  $r$  is the continuously-compounded  $T$ -years interest rate.
4. A *perpetual bond* is a bond with no maturity, i.e., it pays a fixed payment at fixed intervals forever.<sup>1</sup> Under the assumption of constant (continuously compounded) interest rates of 5%, what is the price of a perpetual bond with face value £100 paying annual coupon of 2.50% once a year and whose first coupon payment is due in three months?
5. The prices of zero coupon bonds with face value of £100 and maturity dates in 6, 12 and 18 months are £98, £96 and £93.5 respectively. The price of a 8% bond maturing in 2 years with semiannual coupons is £107. Use the bootstrap method to find the two-year discount factor and spot rates.
6. Consider the following zero-coupon bonds with face value of £100:

Time to maturity (in years)	Annual interest	Bond price (in £)
0.5	0	97.8
1.	0	95.5

Suppose that you are offered by a risk free institution the opportunity to deposit or borrow £1,000,000 in six months for a period of six months earning an interest rate of 5%. Describe in detail an arbitrage opportunity available to you.

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<sup>1</sup>An example of such a bond is the *consol* bond, which was issued in 1749 by the British government to convert a number of outstanding debt issues into a promise to pay 3% interest to the note holders forever. In 1888, the consols were converted from 3% into 2.5% notes so the holder of a £100 consol today receives £2.50 a year in interest.