# MAS362/MAS462/MAS6053 Financial Mathematics Problem Sheet 2 Solutions

1. The farmer would be guaranteed to be paid £300,000 for her harvest if she owned 2000 put options with strike price £150 expiring on December 1st.

2. (a) take a short position in the forward contract,
   (b) borrow £$0.5e^{-0.03\times 0.25}$ for 0.25 years spot interest rate of 3%,
   (c) borrow £$10 - 0.5e^{-0.03\times 0.25}$ for 0.5 years at spot interest rate of 3%,
   (d) buy the asset for £10,
   (e) wait three months,
   (f) collect the 50p dividend and use it to repay first loan, now amounting to 50p,
   (g) wait another three months,
   (h) deliver the asset and collect £9.70,
   (i) use £$(10 - 0.5e^{-0.03\times 0.25}) \times e^{0.03\times 0.5} \approx 9.518$ to repay the loan,
   (j) pocket the difference £$9.70 - (10 - 0.5e^{-0.03\times 0.25}) \times e^{0.03\times 0.5} \approx 0.05263$.

3. 2 call options with strike 10, short in 4 call options with strike 20, 2 call options with strike 30.

4. Let $C_1$ and $C_2$ be the prices of the corresponding American options. We must have $C_1 \geq C_2$, otherwise, if $C_1 < C_2$, we issue the option with expiration at $T_2$, and buy the one expiring at $T_1$ and pocket the difference $C_2 - C_1 > 0$. If at any time $0 \leq t \leq T_2$ the person to whom we issued the option wants to exercise it, we collect £$X$ from her, exercise our option, buy the underlying share for £$X$, and hand the share. If the option we issued is not exercised, we end up with an option on top of our profit of $C_2 - C_1$. We assume that these arbitrage opportunities do not exist, hence we are forced to conclude that $C_1 \geq C_2$.

   As these are call options on non-dividend paying stock, we also know that $C_1 = c_1$ and $C_2 = c_2$, so $c_1 \geq c_2$.

5. (a) This contradicts the inequality $p \leq X$. So use the arbitrage opportunity in the proof: sell the option and wait. If the option is exercised, buy the stock for £3 and pocket £1 plus the stock; if the option is not exercised pocket £4 from the original sale of the option.
   (b) This contradicts the inequality $c \leq S_0$. Sell the call option, buy the stock, pocket 20p and wait. If the option is exercised, deliver your stock, otherwise keep it.

(c) Here ~~$S_0 - Xe^{rT} = 3 - 2e^{-0.025} = 1.05$. So the inequality $c > S_0 - Xe^{rT}$~~ $S_0 - Xe^{-rT} = 3 - 2e^{-0.025} = 1.05$. So the inequality $c > S_0 - Xe^{-rT}$ is violated: the option price should be at least £1.05.

Short sell share for £3, buy call option for 50p, deposit £2.50 for 6 months. After 6 months collect $2.50e^{0.05/2} \approx 2.56$, use option to buy share for £2, and use it to close the short position on the share, pocket £$2.56 - 2 > 0$.

(d) Here $Xe^{-rT} - S_0 = 2e^{-0.025} - 1 = 0.95$, so the inequality $p > Xe^{-rT} - S_0$ is violated (just): the option price should be at least 95p.

Borrow £1.90 for 6 months, buy the share and the option, and wait. After 6 months use the option to sell your share for £2, repay your loan amounting to $1.9e^{0.05/2} \approx 1.95$ and pocket £$2 - 1.95 > 0$.