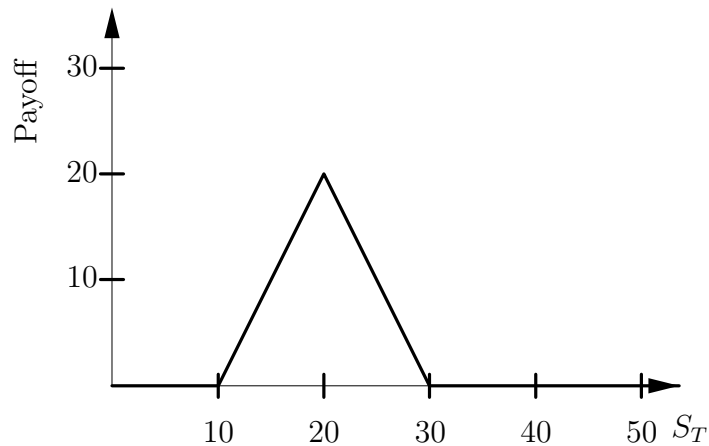


MAS362/MAS462/MAS6051 Financial Mathematics Problem Sheet 2

1. A farmer expects to harvest 2000 tons of wheat in time to deliver it on December 1st this year. The farmer needs to be paid at least £300,000 for the wheat to pay back loans and to prepare for next year's harvest and asks you for advice. You note that Euronext trades options on wheat, each with an underlying asset of one ton of wheat of the same quality as the farmer's. Options are traded with various expirations and strike prices. What is your advise to the farmer assuming she can afford to follow your advise?
2. Consider a six-month forward contract on one share of XYZ Plc. These shares are currently traded for £10 per share, and spot interest rates for all maturities are 3%. Within the next six months, XYZ Plc will pay a single dividend of 50p per share in 3 months. You are given the opportunity to take a short position in this forward contract at a forward price of £9.70. Exhibit in detail an arbitrage opportunity available to you.
3. Describe a portfolio consisting of European call options on a single stock (with different strike prices but the same expiry time T) which has the following payoff function at time T :



(Hint: work from left to right, starting by working out the number of options with strike price 10, and then the number of options with progressively higher strike prices.)

4. Show that, if c_1 and c_2 are the prices of two European call options on the same non-dividend paying stock with price S at time $t = 0$, with the same strike price X and expiry times T_1 and T_2 , respectively, with $T_1 > T_2 > 0$, then $c_1 \geq c_2$. (Hint: consider the corresponding American options.)

5. Assume that the interest rate is 5%, and consider call and put options of both American and European style expiring in 6 months on non-dividend paying stock. For each of the following scenarios check if you can find an arbitrage opportunity and, if you can, describe it:
- (a) The strike price of a European put option is £3 and the option is traded at £4.
 - (b) The shares are traded at £3 and the American call option is traded at £3.20.
 - (c) The shares are traded at £3 and a European call option with strike price of £2 is traded at 50p.
 - (d) The shares are traded at £1 and a European put option with strike price of £2 is traded at 90p.