MAS362/MAS462/MAS6051 Financial Mathematics
Problem Sheet 3

1. A stock currently trades for £25. It is known that S_T, the price of the stock after two months, will be either £23 or £27. Assuming an interest rate of 10% per annum, find the value of a derivative that pays S_T^2 pounds at the end of the two months.

2. A stock (not paying dividends) currently trades for £20. Over each of the next two one-month periods its value is expected to go up by 10% or down by 10%. Assume an interest rate of 12% per annum. Calculate the values of European put and call options with expiry time 2 months and strike price £21.

3. Repeat the previous question, but for American options. When would a rational investor exercise these options?

4. Let X and Y be two random variables whose joint distribution is given by the following table:

	$Y=1$	$Y=2$
$X=-1$	5/36	22/36
$X=1$	7/36	2/36

   i.e., $\mathrm{P}(X=-1, Y=1) = 5/36$, etc.

   (a) Compute the expected values and variances of X and Y.

   (b) Compute the covariance and correlation between X and Y.

   (c) Compute the expected value and variance of $X/3 + 2Y/3 + 1$.

5. Let X be a random variable such that $\log X$ is normally distributed with mean μ and variance σ^2. What is the probability of $X < a$?

6. Let $X_1, \ldots, X_n$ be independent normally distributed random variables and denote the mean of X_i with μ_i and its variance with σ_i^2. What is the distribution of $X_1 + \cdots + X_n$?

7. Consider the partial differential equation

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

   where $f = f(S,t)$, and r and σ^2 are constants.

   Show that this PDE is *linear*, i.e., show that if f_1 and f_2 are solutions of this PDE, then $af_1 + bf_2$ is also a solution for any constants a and b.

   Show that $h(S,t) = kS$, and $h(S,t) = ke^{rt}$, are solutions of this PDE where k is any constant.