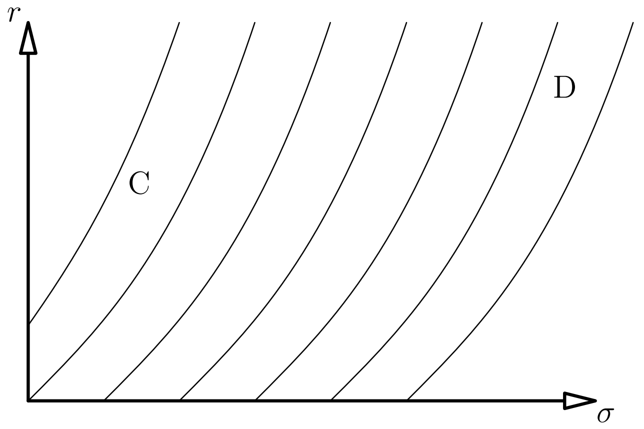
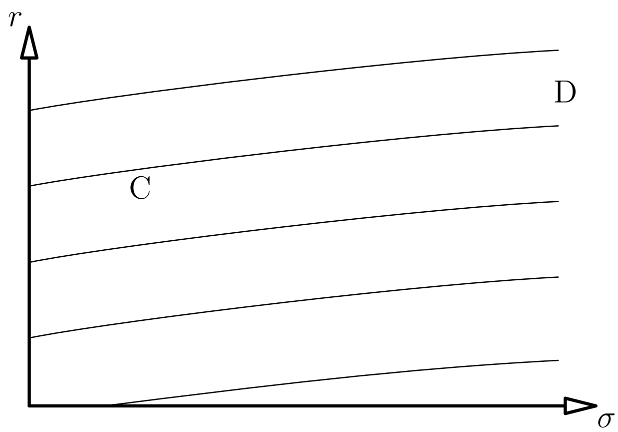
# MAS362/MAS462/MAS6051 Financial Mathematics
# Problem Sheet 5 Solutions

1. The expected returns are 0.275 for C and 0.4 for D, and the standard deviations are 0.130 for C and 0.693 for D.

   This investor would prefer C to D:




   This investor is less risk-averse, and would prefer D to C:




2. None of them can, given our axioms on investors' preferences. A, B and C all have decreasing segments of curve, which is impossible (as explained in the lectures). In D, as the two lines cross they must in fact have the same attractiveness (otherwise the crossing point has two different attractiveness) so two different values of the average return would be equally attractive for the same risk, which is implausible.

3. You would choose an efficient portfolio, consisting of lending £$t$ and investing £$(1000 - t)$ in the market portfolio. The expected return is

$$0.05t + 0.2(1000 - t) = 200 - 0.15t,$$

which will be 1000 when $t = -16000/3$. So the desired investment is to borrow £16000/3 and to invest £19000/3 in the market portfolio. The standard deviation is £(0.1(19000/3)) =£633.3. (So ending up with a large debt is not unlikely.)

4. The capital market line is given by

$$r - 0.05 = \frac{0.1 - 0.05}{0.15}\sigma,$$

i.e. $r = \sigma/3 + 0.05$. The beta coefficient of stock 1 is

$$\beta_1 = \frac{\text{Covar(Stock 1}, M)}{\sigma_M^2} = \frac{\rho(\text{Stock 1}, M)\sigma_1}{\sigma_M} = \frac{0.9 \times 0.2}{0.15} = 1.2.$$

We obtain the expected return using the security market line

$$r_1 = r_B + (r_M - r_B)\beta_1 = 0.05 + (0.1 - 0.05) \times 1.2 = 0.11,$$

i.e. 11%.

The beta coefficient of stock 2 must satisfy $r_2 = r_B + (r_M - r_B)\beta_2$ which gives $\beta_2 = 0.8$ and so its correlation with the market portfolio is $0.8(\sigma_M/\sigma_2) = 2/3$.

Similarly, the beta coefficient of stock 3 is $\beta_3 = 0.4$, and

$$\beta_3 = \frac{\rho(\text{Stock 3}, M)\sigma_3}{\sigma_M}$$

implies $\sigma_3 = 0.086$, i.e. 8.6%.

5. Portfolios entirely consisting of the risky investments consist of $t$ units of A and $1 - t$ units of B, for some $t$. Letting $r_t$ and $\sigma_t$ be the expected return and standard deviation respectively, $r_t = 0.1t + 0.15(1 - t) = 0.15 - 0.05t$ and

$$\sigma_t = \sqrt{(0.1t)^2 + 2t(1 - t)(0.5)(0.1)(0.2) + (0.2(1 - t))^2} = \sqrt{0.03t^2 - 0.06t + 0.04}.$$

So the function $\frac{r_t - r_B}{\sigma_t}$ we need to maximise is

$$f(t) = \frac{0.1 - 0.05t}{\sqrt{0.03t^2 - 0.06t + 0.04}}.$$

Differentiating and solving $f'(t) = 0$ gives $t = 2/3$, so the market portfolio is 2/3 units invested in A and 1/3 in B.

For (b), to find a portfolio with minimal standard deviation for given expected return we know we can restrict ourselves to the capital market line, i.e. invest $s$ units risk-free and $1 - s$ units in the market portfolio.

The expected return of the market portfolio is $2/3(0.1) + 1/3(0.15) = 7/60$, and so the expected return of our portfolio is $r'_s = 0.05s + (7/60)(1-s) = 7/60 - s/15$. Solving $r'_s = 0.12$ gives $s = -1/20$. So we borrow 1/20 units and invest 21/20 in the market portfolio (14/20 in A and 7/20 in B, given the details of the market portfolio already found). In this case, one unit is £10,000,000, so we borrow £500,000, giving £10,500,000 to invest, £7,000,000 in A and £3,500,000 in B.

6. The expected return $r$ satisfies $r = 0.04 - (0.12 - 0.04) \times 1/2 = 0$.

   People invest in this zero-expected-return investment together with the risky investments: the fact that $\beta < 0$ implies that the returns of this investment and those of the market are negatively correlated thus this asset reduces the exposure to market risk.